Statistics and Mechanics Year 2

Original practice check

Three short checks on the normal distribution, projectiles and moments. A calculator may be used. Take gravitational acceleration as 9.8 metres per second squared. Attempt the questions before reading the solutions. These original questions sample selected skills; they are not an official Edexcel paper.

1. Normal distribution

X is normally distributed with mean 50 and standard deviation 4. Find $P(X > 54)$, giving your answer to three significant figures.

2. Projectiles

A particle is projected from level ground at $20\,\mathrm{m\,s^{-1}}$, at 30° above the horizontal. Neglect air resistance and take $g = 9.8\,\mathrm{m\,s^{-2}}$. Find its time of flight and horizontal range when it returns to the same level.

3. Moments and equilibrium

A uniform horizontal beam AB is 4 m long and weighs 40 N. It is supported at A and B. A further downward load of 20 N acts 1 m from A. Find the upward reactions at the two supports.

Find the mistake

Events A and B satisfy $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$. A student writes $P(A \mid B) = P(A) = 0.4$. Explain why this is wrong and find the correct conditional probability.

Further practice: TMUA papers and solutions and maths topic guides.

Hints and worked solutions

1. Normal distribution

Hint: Standardise using the standard deviation, then choose the upper tail.

With $Z = (X - 50)/4$, the threshold $X = 54$ corresponds to $Z = 1$.

Hence $P(X > 54) = P(Z > 1) = 1 - \Phi(1) \approx 0.158655$, where Φ is the standard normal cumulative distribution function.

Answer: $1 - \Phi(1) \approx 0.159$ (three significant figures).

2. Projectiles

Hint: Resolve the initial velocity into components. Use zero vertical displacement to find the non-zero flight time.

The initial horizontal and vertical velocity components are $20\cos 30^\circ = 10\sqrt{3}\,\mathrm{m\,s^{-1}}$ and $20\sin 30^\circ = 10\,\mathrm{m\,s^{-1}}$.

Taking upward as positive, vertical displacement is $y = 10t - 4.9t^2$. At landing, $y = 0$; the non-zero solution is $t = 10/4.9 = 100/49\,\mathrm{s} \approx 2.04\,\mathrm{s}$.

Horizontal velocity is constant, so the range is $(10\sqrt{3})(100/49) = 1000\sqrt{3}/49\,\mathrm{m} \approx 35.3\,\mathrm{m}$.

Answer: Time $\dfrac{100}{49}$ s; range $\dfrac{1000\sqrt{3}}{49}$ m (approximately 2.04 s and 35.3 m).

3. Moments and equilibrium

Hint: The beam's weight acts at its midpoint. Taking moments about one support eliminates its reaction.

Let the upward reactions be R_A and R_B. Vertical equilibrium gives $R_A + R_B = 40 + 20 = 60\,\mathrm{N}$.

Taking moments about A, $4R_B = 40(2) + 20(1) = 100$. Therefore $R_B = 25\,\mathrm{N}$.

It follows that $R_A = 60 - 25 = 35\,\mathrm{N}$.

Answer: $R_A = 35\,\mathrm{N}$, $R_B = 25\,\mathrm{N}$.

Find the mistake: correction

$P(A \mid B) = P(A \cap B)/P(B) = 0.3/0.5 = 0.6$.

The student's equality would hold for independent events. Here $P(A)P(B) = 0.4(0.5) = 0.2$, which differs from $P(A \cap B) = 0.3$; the events are not independent.

What to practise next

1. For normal probabilities, draw and shade the required tail before entering values; distinguish variance from standard deviation.
2. For projectiles, keep separate horizontal and vertical equations linked by the same time.
3. For equilibrium, draw every force at its point of application and choose a moment centre that removes an unknown.