Pure Mathematics Year 2

Original practice check

Three short checks on trigonometric equations, parametric tangents and integration by parts. Angles are in radians. Attempt the questions before reading the solutions. These original questions sample selected skills; they are not an official Edexcel paper.

1. Trigonometric equations

Solve $2\sin^2 x + \sin x - 1 = 0$ for $0 \le x < 2\pi$.

2. Parametric differentiation

A curve is given by $x = t^2 + 1$, $y = t^3 - 3t$. Find the equation of its tangent at $t = 2$.

3. Integration by parts

Evaluate $\displaystyle\int_0^1 xe^x \,\mathrm{d}x$.

Find the mistake

A student calculates $\int_{-1}^{1} x \,\mathrm{d}x = 0$ and concludes that the total area between $y = x$ and the horizontal axis on this interval is zero. Correct the conclusion.

Further practice: TMUA papers and solutions and maths topic guides.

Hints and worked solutions

1. Trigonometric equations

Hint: Treat $\sin x$ *as the variable while factorising, then find every angle in the stated interval.*

$2\sin^2 x + \sin x - 1 = (2\sin x - 1)(\sin x + 1)$. Hence $\sin x = \frac{1}{2}$ or $\sin x = -1$.

In the given interval, $\sin x = \frac{1}{2}$ gives $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$. The equation $\sin x = -1$ gives $x = \frac{3\pi}{2}$.

Answer: $x = \dfrac{\pi}{6},\ \dfrac{5\pi}{6},\ \dfrac{3\pi}{2}$.

2. Parametric differentiation

Hint: Use $\mathrm{d}y/\mathrm{d}x = (\mathrm{d}y/\mathrm{d}t)/(\mathrm{d}x/\mathrm{d}t)$*, and find the point as well as the gradient.*

At $t = 2$, $x = 5$ and $y = 2$, so the required point is $(5, 2)$.

$\mathrm{d}x/\mathrm{d}t = 2t$ and $\mathrm{d}y/\mathrm{d}t = 3t^2 - 3$. Thus $\mathrm{d}y/\mathrm{d}x = (3t^2 - 3)/(2t)$, which is $9/4$ at $t = 2$.

The tangent through $(5, 2)$ with gradient $9/4$ is $y - 2 = \frac{9}{4}(x - 5)$.

Answer: $y - 2 = \dfrac{9}{4}(x - 5)$.

3. Integration by parts

Hint: In integration by parts, differentiate x *and integrate* e^x*.*

Take $u = x$ and $\mathrm{d}v = e^x\,\mathrm{d}x$, so $\mathrm{d}u = \mathrm{d}x$ and $v = e^x$.

$$\begin{aligned}\int_0^1 xe^x\,\mathrm{d}x &= [xe^x]_0^1 - \int_0^1 e^x\,\mathrm{d}x \\ &= e - (e - 1) = 1.\end{aligned}$$

Answer: 1.

Find the mistake: correction

The integral is correctly calculated as zero: the contribution below the axis is $-1/2$ and the contribution above the axis is $1/2$.

Total area adds their magnitudes. Equivalently, $\int_{-1}^{1} |x|\,\mathrm{d}x = \frac{1}{2} + \frac{1}{2} = 1$.

What to practise next

1. Keep the interval and angle units visible when solving trigonometric equations; use a sketch to check for missing roots.
2. Before integrating, decide whether simplification, substitution or integration by parts makes the structure easier.
3. Use graph sketches to check signs, turning points and cancellation before committing to lengthy algebra.